SOME PROBLEMS IN THE THEORY OF VISCOUS JETS

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Self-similar solutions are known [1-3] for problems on the propagation of laminar, plane (free and semibounded) jet, e.g., source in heated space. Relations obtained in these works have a significant disadvantage: a singularity arises as $x \to 0$. This means that it is possible to use these equations to correctly describe the flow only at large values of x. In the process of finding non-self-similar solutions for jet issuing from a finite orifice, the form of the stream function has been established and the first three terms of the series (dynamic problem) have been obtained in an analytic form [4]. Non-self-similar thermal problems are considered in [5, 6] where these studies [4] have been extended. However, the question of the determination of constants of integration and the region of convergence of the obtained series remained open in [4-6]. The solution to the problem on the propagation of laminar, plane (free and semibounded) jet in heated space is obtained in the present paper for different boundary conditions of the temperature of the surrounding medium or the surface for arbitrary Prandtl numbers.

1. Laminar boundary-layer equations for stationary, plane, incompressible fluid flow at constant pressure in the external flow have the form [3]:

$$u\partial u/\partial x + v\partial u/\partial y = \nu\partial^2 u/\partial y^2,$$
$$\partial u/\partial x + \partial v/\partial y = 0,\ u\partial\Delta T/\partial x + v\partial\Delta T/\partial y = a\partial^2\Delta T/\partial y^2. \tag{1.1}$$

The boundary conditions for free plane jet are written in the following form:

$$v = \partial u/\partial y = 0 \text{ at } y = 0,\ u \to 0 \text{ as } y \to \infty; \tag{1.2}$$

$$\partial\theta/\partial y|_{y=0} = 0,\ \ \theta(x, \pm\infty) = 0,\ \theta = \Delta T; \tag{1.3}$$

$$\theta(x, +\infty) = 1,\ \theta(x, -\infty) = 0,\ \theta = (T - T_2)/(T_1 - T_2). \tag{1.4}$$

For plane, semibounded jet,

$$v = u = 0 \text{ at } y = 0,\ u \to 0 \text{ as } y \to \infty; \tag{1.5}$$

$$\theta(x, 0) = 0,\ \theta(x, \infty) = 0,\ \theta = \Delta T; \tag{1.6}$$

$$\partial\theta/\partial y|_{y=0} = 0,\ \theta(x, \infty) = 0,\ \theta = \Delta T; \tag{1.7}$$

$$\theta(x, 0) = 1,\ \theta(x, \infty) = 0,\ \theta = (T - T_\infty)/(T_w - T_\infty); \tag{1.8}$$

$$\partial\theta/\partial y|_{y=0} = -1,\ \theta(x, \infty) = 0,\ \theta = k(T - T_\infty)/q_w. \tag{1.9}$$

In order to determine nontrivial solution of the system (1.1) at zero boundary conditions it is necessary to specify integral conditions which are obtained by integrating equations of motion and thermal energy, taking into account continuity equation and boundary conditions [1-3]:

$$\int\limits_{-\infty}^{+\infty} \rho u^2 dy = K_0,\quad \int\limits_0^\infty u^2\left(\int\limits_0^y u\,dy\right)dy = E_0; \tag{1.10}$$

$$\int\limits_{-\infty}^{+\infty} \rho u\Delta i\,dy = Q_0,\quad \frac{d}{dx}\int\limits_0^\infty u\Delta T\left(\int\limits_0^y u\,dy\right)dy = -a\int\limits_0^\infty u\frac{\partial\Delta T}{\partial y}\,dy. \tag{1.11}$$

Here u and v are velocity components; $\Delta T = T - T_\infty$, excess temperature; x and y, streamwise and transverse coordinates; ν, a, and k, coefficients of kinematic viscosity, thermal diffu-

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sivity, and thermal conductivity; ρ, density; q, heat flux; T_∞ and T_W, temperatures of the surrounding fluid at rest and the wall; T_1 and T_2, temperatures outside the jet at $y = +\infty$ and $y = -\infty$, respectively.

2. The previously obtained solutions to the system of equations (1.1) were based on the following similarity transformations:

$$\psi(x, \eta) = \nu^{\beta+1} f_0(\eta) x^{\beta+1}, \ \theta(x, \eta) = d_0(\eta) x^\delta,$$
$$x = x, \ \eta = (x\nu)^\beta y,$$

which lead to the limitations mentioned above. Consider a more general similarity transformation:

$$\psi(x, \eta) = \nu^{\beta+1} f(\eta) p(x), \ \theta(x, \eta) = d(\eta)\varepsilon(x),$$
$$x = x, \ \eta = \nu^\beta \sigma(x) y. \tag{2.1}$$

Substituting (2.1) in (1.1), we obtain

$$\begin{gathered} f_{\eta\eta\eta} p\sigma^3 + f f_{\eta\eta} p\sigma^2 p_x - f_\eta^2 p\sigma (p\sigma)_x = 0, \\ \frac{1}{\Pr} d_{\eta\eta}\sigma^2\varepsilon + f d_\eta \sigma\varepsilon p_x - f_\eta d p\sigma\varepsilon_x = 0. \end{gathered} \tag{2.2}$$

The solution to Eqs. (2.2) is written in the form

$$p = X^{\beta+1}, \ \sigma = X^\beta, \ \varepsilon = X^\delta, \ X = x(1 + \gamma/((\beta + 1)x)). \tag{2.3}$$

Here the form of the resulting functions f(η) and d(η) satisfying boundary and integral conditions coincides with known solutions for free ($\beta = -^2/_3$, $\delta = -^1/_3$, $\delta = 0$) [1, 3] and semi-bounded jets [$\beta = -^3/_4$, $\delta = -(3\Pr + 1)/8\Pr$, $\delta = -^1/_4$, $\delta = 0$, $\delta = ^3/_4$] [1, 2, 7]. Further, we find

$$u = \nu^{2\beta+1} f' X^{2\beta+1}, \ \theta = dX^\delta, \ v = -\nu^{\beta+1}[(\beta + 1)f + \beta f'\eta] X^\beta. \tag{2.4}$$

It is possible to observe that at large x, Eqs. (2.4) transform to expressions from [1-3]. Without writing out known solutions for plane jet sources, we observe only that Eqs. (2.4) also differ from [1-3] in the variable η:

$$\eta = (x\nu)^\beta (1 + \gamma/(\beta + 1)x)^\beta y.$$

3. Following [4-6], approximate solution to the problem is sought in the form of the series:

$$\psi = \nu^{\beta+1} \sum_{i=0}^{\infty} f_i(\eta)\, x^{\lambda_i}, \quad \theta = \sum_{i=0}^{\infty} d_i(\eta)\, x^{\delta-i},$$
$$\eta = (x\nu)^\beta y, \ \lambda_i = \beta + 1 - i. \tag{3.1}$$

Substituting (3.1) in equations from system (1.1) and equating coefficients of same powers in x, we get an infinite system of differential equations for the determination of unknown functions:

$$\begin{gathered} f_0''' + (\beta + 1) f_0 f_0'' - (2\beta + 1) f_0'^2 = 0, \\ \frac{1}{\Pr} d_0'' + (\beta + 1) f_0 d_0' - \delta f_0' d_0 = 0, \\ f_i''' + (\beta + 1) f_0 f_i'' - (4\beta + 2 - i) f_0' f_i' + \lambda_i f_0'' f_i = N_i, \\ \frac{1}{\Pr} d_i'' + (\beta + 1) f_0 d_i' - (\delta - i) f_0' d_i = M_i, \\ N_i = \sum_{j=1}^{i-1} \{[2\beta + 1 - (i - j)] f_j' f_{i-j}' - (\beta + 1 - j) f_j f_{i-j}''\}, \\ M_i = \delta f_i' d_0 - \lambda_i f_i d_0' + \sum_{j=1}^{i-1} \{[\delta - (i - j)] f_j' d_{i-j} - (\beta + 1 - j) f_j d_{i-j}'\}. \end{gathered} \tag{3.2}$$

Here prime denotes differentiation with respect to η. The first two equations in (3.2) have known solutions [1-3, 7]. In order to obtain higher order terms we turn to the third equation in (3.2), in which it is expedient to introduce new variables [4]:

$$f_i = f_0' \int \frac{y_i}{f_0'} d\eta, \quad w_i = z y_i, \quad t = z^3.$$

After a number of transformations not included here, it reduces to the Legendre equation ($\beta = -2/3$)

$$(1 - z^2) y_i'' - 2 z y_i' + 6 i y_i = \frac{N_i}{\alpha^2 (1 - z^2)} \tag{3.3}$$

and to the hypergeometric equation ($\beta = -3/4$)

$$t(1 - t) w_i'' + \left(\frac{1}{3} - \frac{4}{3} t\right) w_i' + \frac{2 + 8i}{3} w_i = \frac{144 N_i}{9\alpha^2 (1 - t)}. \tag{3.4}$$

The following self-similar equation [2, 3] is used here:

$$f_0 = 6\alpha z, \; z = \operatorname{th} \alpha\eta, \; \alpha = (K_0/48\rho)^{1/3},$$

$$f_0 = \alpha z^2, \quad \eta = \frac{12}{\alpha} \int \frac{dz}{1 - z^3}, \quad \alpha = (40 E_0)^{1/4}.$$

The solutions to Eqs. (3.3) and (3.4) may be expressed in the form of the sum of solutions of the corresponding homogeneous differential equation and particular integral φ_i. The following equations are obtained using boundary conditions (1.2) and (1.5), respectively,

$$f_i = c_i (1 - z^2) \int \frac{P_k(z)}{(1 - z^2)^2} dz + \varphi_i;$$

$$f_i = c_i z (1 - z^3) \int \frac{F(a + 2/3, \, b + 2/3, \, 5/3, \, z^3)}{(1 - z^3)^2} dz + \varphi_i,$$

$$ab = -2(1 + 4i)/3, \; a + b = 1/3.$$

Here $P_k(z)$ is Legendre polynomial of the first kind [8] limited to the interval $-1 \le z \le 1$ with conditions $6i = k(k + 1)$, $k = 0, 1, 2, \ldots$, $i = 1, 2, \ldots$ (consequently, $i = 1$, $k = 2$ [4]; $i = 7$, $k = 6$, etc.); $F(a + 2/3, b + 2/3, 5/3, z^3)$ is hypergeometric function which can be expressed in terms of a polynomial [8] when the following identity is satisfied:

$$5/6 - (\sqrt{25 + 96i})/6 = -k \tag{3.5}$$

$i = 1$, $k = 1$ [4] correspond to the first value of i according to (3.5), $i = 29$, $k = 8$ correspond to the second value of i, etc. We note that the constant of integration passes through the integral condition which, in terms of the variable η, has the form

$$\int_{-\infty}^{+\infty} \left(2 f_0' f_i' + \sum_{j=1}^{i-1} f_j' f_{i-j}'\right) d\eta = 0,$$

$$\int_0^{\infty} \left(f_i f_0'^2 + \sum_{j=0}^{i-1} f_j \sum_{k=0}^{i-j} f_k' f_{i-j-k}'\right) d\eta = 0.$$

This indicates that the indeterminate constants will continue to appear in the series (3.1). The particular integral for the given equation can be expressed in the form

$$\varphi_{i+1} = \frac{\gamma}{(i+1)(\beta+1)} \left[\beta f_i' \eta + (\beta + 1 - i) f_i\right], \quad c_1 = \gamma.$$

It is then possible to sum these up to write asymptotic expressions for ψ and u:

$$\psi = \nu^{\beta+1} \left\{ f_0 x^{\beta+1} + \frac{\gamma}{\beta+1} \sum_{i=0}^{n} \frac{1}{i+1} (\beta f_i' \eta + \lambda_i f_i) x^{\beta - i} + \ldots \right\}. \tag{3.6}$$

$$u = \nu^{2\beta+1} \left\{ f_0' x^{2\beta+1} + \frac{\gamma}{\beta+1} \sum_{i=0}^{n} \frac{1}{i+1} (\beta f_i'' \eta + (\lambda_i + \beta) f_i') x^{2\beta - i} + \ldots \right\}.$$

Here $n = 6$ for the free jet and $n = 28$ for the semibounded jet. The solution for temperature is also obtained in a similar manner:

$$\theta = d_0 x^\delta + \frac{\gamma}{\beta+1} \sum_{i=0}^{n} \frac{1}{i+1} \left(\beta d_i' \eta + (\delta - i) d_i\right) x^{\delta-1-i} + \ldots \tag{3.7}$$

In principle, it is possible to approximate ψ, u, and θ by higher-order terms ($i \geq 7$, $i \geq 29$) but in doing this new undetermined constants appear and the resulting equations have an extremely unwieldy form.

Continuing the analysis, we observe that Eqs. (3.6) and (3.7), representing approximate non-self-similar solution to the problem, following directly from Eqs. (2.4) if they are written in the form

$$u = \nu^{2\beta+1} f_0' (\eta (X/x)^\beta) X^{2\beta+1}, \quad \theta = d_0 (\eta (X/x)^\beta) X^\delta$$

and expanded in series

$$u(t) = u(0) + tu'(0) + \frac{1}{2!} t^2 u''(0) + \ldots$$

in terms of the variable $t = \gamma/(\beta + 1)x$

$$u = \nu^{2\beta+1} \left\{ f_0' x^{2\beta+1} + \frac{\gamma}{\beta+1} \sum_{i=0}^{\infty} \frac{1}{i+1} \left(\beta f_i'' \eta + (\lambda_i + \beta) f_i'\right) x^{2\beta-i} \right\}, \tag{3.8}$$

$$\theta = d_0 x^\delta + \frac{\gamma}{\beta+1} \sum_{i=0}^{\infty} \frac{1}{i+1} \left(\beta d_i' \eta + (\delta - i) d_i\right) x^{\delta-1-i}.$$

These results (3.8) make it possible to conclude that the new undetermined constants that appear in series ($i \geq 7$, $i \geq 29$) according to (3.6) and (3.7), do not correspond to the value of the quantity γ specified a priori. These, apparently, represent the general shortcomings of asymptotic boundary-layer schemes. In addition, it is possible to observe that the series (3.6) and (3.7) converge to exact solutions (2.4) for all

$$|\gamma/(\beta + 1)x| < 1.$$

In order to complete the solution to the problem (1.1)-(1.11), it is necessary to determine the constant of integration γ which is present in boundary and integral conditions. It is characterized by mass flow rate per second through the initial jet section m_0:

$$\gamma = m_0^3/(108\rho^2 \nu K_0), \quad \gamma = m_0^4/(160\rho^4 \nu E_0). \tag{3.9}$$

Developing analysis further, we note that the relations (2.3), (2.4), and (3.9) are self-similar solutions [1-3] but displaced along the x axis. Physically, this means that the jet issues from a fictitious source located within the nozzle at such a distance from the exit section that the fluid flow rate at the nozzle section given by self-similar solution coincides with actual flow rate.

4. Let us consider again the above solution for the temperature field. The energy equation was studied while specifying integral conditions for excess enthalpy. It replaces detailed conditions for the flow from the nozzle. However, actual jets (issuing from finite-sized nozzles) can have different initial velocity (temperature) profile shapes. Hence it is more interesting, apparently, to consider the temperature distribution at a certain jet section. The solution to the energy equation in such a formulation may allow, in particular, qualitative and quantitative description of the process of distortion of the initial temperature profile, determination of the effect of Pr, etc.

In this case, the following equation is specified in addition to Eqs. (1.1), boundary (1.2), (1.3), (1.5)-(1.7), and integral conditions (for the velocity field

$$\Delta T(x, y)|_{x=x_0} = \Delta T(x_0, y) = T_0. \tag{4.1}$$

Here self-similarity, in general, is not present and it is necessary to solve the eigenvalue and eigenfunction problem. The solution to the third equation of the system (1.1) is sought in the form

$$\Delta T(x, \eta) = \sum_{i=0}^{\infty} c_i X^{\alpha_i} \theta_i(\eta). \tag{4.2}$$

Using the above results (4.2) to determine $\theta_i(\eta)$ we have

$$\frac{1}{\mathrm{Pr}} \theta_i'' + (\beta + 1) f\theta_i' - \alpha_i f' \theta_i = 0, \tag{4.3}$$

which reduces to hypergeometric equation for the given boundary conditions (1.3) with the help of the transformation $t = \tanh^2 \alpha\eta$

$$t(1-t)\theta_i'' + \left[\frac{1}{2} + \left(\mathrm{Pr} - \frac{3}{2}\right)t\right]\theta_i' - \frac{3}{2}\mathrm{Pr}\,\alpha_i\theta_i = 0, \tag{4.4}$$
$$\theta_i(1) = 0, \quad \lim_{t\to 0} t^{1/2}\theta_i' = 0.$$

The solution to Eq. (4.4) can be written in the form

$$\theta_i = c_1 F(a, b, 1/2, t) + c_2 \sqrt{t} F(a + 1/2, b + 1/2, 3/2, t), \tag{4.5}$$
$$a + b = 1/2 - \mathrm{Pr}, \ ab = 3\alpha_i \mathrm{Pr}/2.$$

The second boundary condition (4.4) shows that the first integral expression (4.5) is the required solution which satisfies the first boundary condition (4.4) when $a = i + {}^1/_2$. Then

$$\alpha_i = -(2i + 1)(\mathrm{Pr} + i)/3\mathrm{Pr}. \tag{4.6}$$

Keeping in view Eqs. (4.2), (4.5), and (4.6), we find

$$\Delta T = \sum_{i=0}^{\infty} C_i (X/X_0)^{\alpha_i} (1-t)^{\mathrm{Pr}} F(-i, i + \mathrm{Pr} + 1/2, 1/2, t). \tag{4.7}$$

Further, since Jacobian polynomials [9] are orthogonal, the constant C_i is determined by:

$$C_i = \frac{(2i + \mathrm{Pr} + 1/2)\,\Gamma(i + \mathrm{Pr} + 1/2)\,\Gamma(i + 1/2)}{\pi\Gamma(i+1)\,\Gamma(\mathrm{Pr} + i + 1)} \int_0^1 T_0 t^{-1/2} F(-i, i + \mathrm{Pr} + 1/2, 1/2, t)\, dt. \tag{4.8}$$

In deriving (4.8), relation between hypergeometric functions and Jacobian polynomials [9] is used:

$$P_i^{\alpha,\beta}(\xi) = \frac{(i+\alpha)!}{\alpha!\, i!} F\left(-i, i + \alpha + \beta + 1, \alpha + 1, \frac{1}{2} - \frac{1}{2}\xi\right).$$

The above theory can be generalized even for the case of semibounded jet with boundary conditions (1.6), i.e., "similar" boundary conditions for velocity and temperature, and insulated plate condition (1.7).

Equation (4.3) in variable (2.1) reduces to hypergeometric equation:

$$t(1-t)\theta_i'' + \left[\frac{2}{3} + \left(\mathrm{Pr} - \frac{5}{3}\right)t\right]\theta_i' - \frac{8}{3}\alpha_i \mathrm{Pr}\,\theta_i = 0;$$
$$1)\ \theta_i(0) = 0, \quad \theta_i(1) = 0; \qquad 2)\ \lim_{t\to 0} t^{2/3}\theta_i' = 0, \quad \theta_i(1) = 0.$$

Here the prime denotes differentiation with respect to $t = z^3$. The desired results are written in the form

$$1)\ \Delta T = \sum_{i=0}^{\infty} C_i (X/X_0)^{\alpha_i} t^{1/3} (1-t)^{\mathrm{Pr}} F(-i, i + \mathrm{Pr} + 4/3, 4/3, t); \tag{4.9}$$
$$2)\ \Delta T = \sum_{i=0}^{\infty} C_i (X/X_0)^{\alpha_i} (1-t)^{\mathrm{Pr}} F(-i, i + \mathrm{Rr} + 2/3, 2/3, t);$$
$$1)\ \alpha_i = -\frac{(i+1)(1 + 3\,\mathrm{Pr} + 3i)}{8\,\mathrm{Pr}}; \qquad 2)\ \alpha_i = -\frac{(3i+2)(\mathrm{Pr} + i)}{8\,\mathrm{Pr}}.$$

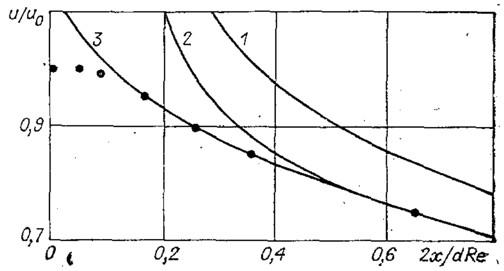

Fig. 1

Coefficients C_i are determined from orthogonality conditions for Jacobian polynomials:

$$1)\ C_i = A_i \int_0^1 T_0 F(-i, i + \Pr + 4/3, 4/3, t)\, dt;$$

$$2)\ C_i = B_i \int_0^1 T_0 t^{-1/3} F(-i, i + \Pr + 2/3, 2/3, t)\, dt;$$

$$A_i = \frac{(2i + \Pr + 4/3)\,\Gamma(i + \Pr + 4/3)\,\Gamma(i + 4/3)}{[\Gamma(4/3)]^2\,\Gamma(i+1)\,\Gamma(i + \Pr + 1)};$$

$$B_i = \frac{(2i + \Pr + 2/3)\,\Gamma(i + \Pr + 2/3)\,\Gamma(i + 2/3)}{[\Gamma(2/3)]^2\,\Gamma(i+1)\,\Gamma(i + \Pr + 1)}.$$

The expression for T_0 is given in Eq. (4.1). We observe that the principal term in series (4.2), as seen from results (4.7) and (4.9), is given by expressions which coincide with solutions (2.4) accurate to the constant, obtained above in providing initial condition for integral relation (1.10) and (1.11) (for the temperature field). The following terms of series (4.2) make it possible to take into account the effect of initial temperature profile T_0, while Prandtl number Pr has significant effect on subsequent asymptotics. It is interesting that the constant $\bar{c}_1$ in the self-similar solution [6] is not determined by integral condition [when $\beta = -^3/_4$, $\delta = -(3\Pr + 1)/8\Pr$], as already mentioned in [1], except in the case Pr = 1, when there is a general (not associated with the assumption of self-similarity) invariant. If, instead of the integral condition, relation (4.1) is introduced, then the problem, as seen from the results obtained, is completely solved.

The variation of maximum velocity along the axis of the free jet is shown in the figure (u_0 is jet velocity at the initial section, d is the nozzle diameter, $Re = u_0 d/2\nu$ is the Reynold's number, $x_* = 2x/dRe$ is the nondimensional axial coordinate [10]). The curve 3 represents computations using Eqs. (2.4) and the points represent numerical solution [10]. A comparison of velocity distribution u/u_0 computed from analytical expressions ($\gamma_0 = 4\nu\gamma/u_0 d^2 = 0.0867$) with numerical results [10], indicated their good agreement, except in the region close to 0 where Eqs. (2.4) give higher, though not contradictory, values. Self-similar (curve 1) and approximate non-self-similar [4] [three terms of the series (3.6)] (curve 2) analytical solutions are also shown in the figure.

In conclusion, we observe that all the approximate non-self-similar solutions found earlier are obtained as a particular case from results of the present work.

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COLLISION OF PLANE, VISCOUS, MULTILAYERED JETS

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UDC 532.522+532.526

In order to determine the differences in real flow with high-speed collision of metallic plates from known [1, 2] inviscid flow, Rubtsov [3] considered the problem of symmetric impingement of plane viscous jets with free boundary. The problem is solved approximately assuming boundary-layer corrections to inviscid flow near the free boundaries at sufficiently large Reynolds numbers. A solution is obtained to the first approximation from simplified correction $w(\varphi, \psi)$ to the inviscid velocity $u_0(\varphi, \psi)$ along the stream line. The simplified equation is obtained from Navier–Stokes equations by carrying out order-of-magnitude analysis. It is of interest to use this method to study the problem of jet collision when each jet comprises a number of layers with different viscosity but the same density.

1. Consider stationary inviscid flow in the region shown in Fig. 1. Two jets of equal thickness h flow from infinity with the same velocity U at an angle γ to the axis of symmetry. The x axis is along the axis of symmetry. Consider half the flow region. The free jet consists of N layers of equal density ρ and different viscosity μ_l and thickness δ_l, $l = 1, 2, \ldots, N$, $\sum_{l=1}^{N} \delta_l = h$. The flow region is limited by the x axis and two free boundaries Σ_1 and Σ_2. There are $N-1$ boundaries in the flow region $\Gamma_1, \Gamma_2, \ldots, \Gamma_{N-1}$. The velocity components along x and y are denoted by u and v. Normalizing x and y by h, u, v by U, and pressure p by ρU^2, Navier–Stokes equations are written in the form

$$u_l \frac{\partial u_l}{\partial x} + v_l \frac{\partial u_l}{\partial y} = -\frac{\partial p_l}{\partial x} + \frac{1}{\mathrm{Re}_l} \Delta u_l, \qquad (1.1)$$
$$u_l \frac{\partial v_l}{\partial x} + v_l \frac{\partial v_l}{\partial y} = -\frac{\partial p_l}{\partial y} + \frac{1}{\mathrm{Re}_l} \Delta v_l,$$

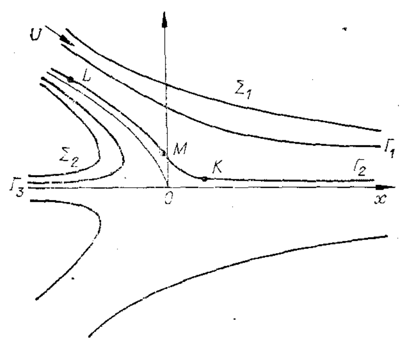

Fig. 1

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